Mini-Course on Market Design: Matching, Carpooling, and Congestion Pricing

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IWGTEA 2026
São Paulo, Brazil

Lecture 1: Two-sided matching with transfers

Lecture 2: Vickrey (1969) model of traffic congestion. Congestion pricing, carpooling, and commuter welfare.

Lecture 3: Bringing it all together: self-driving cars, carpooling, and congestion pricing.

Lecture 4: Empirics: Congestion Pricing in New York City

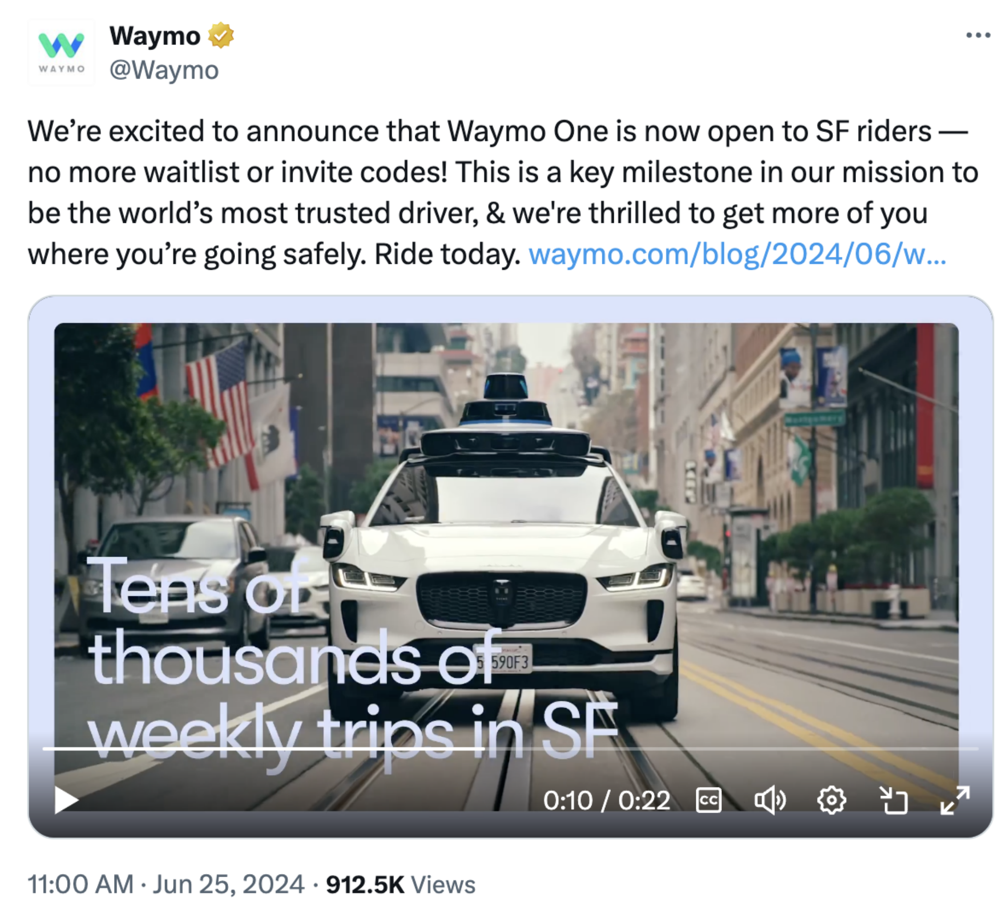
Three emerging technologies that will transform transportation:

- Self-driving cars
- Frictionless time-dependent tolls
- Convenient, efficient carpooling

What will a market powered by these technologies look like?

How should it be designed and organized?

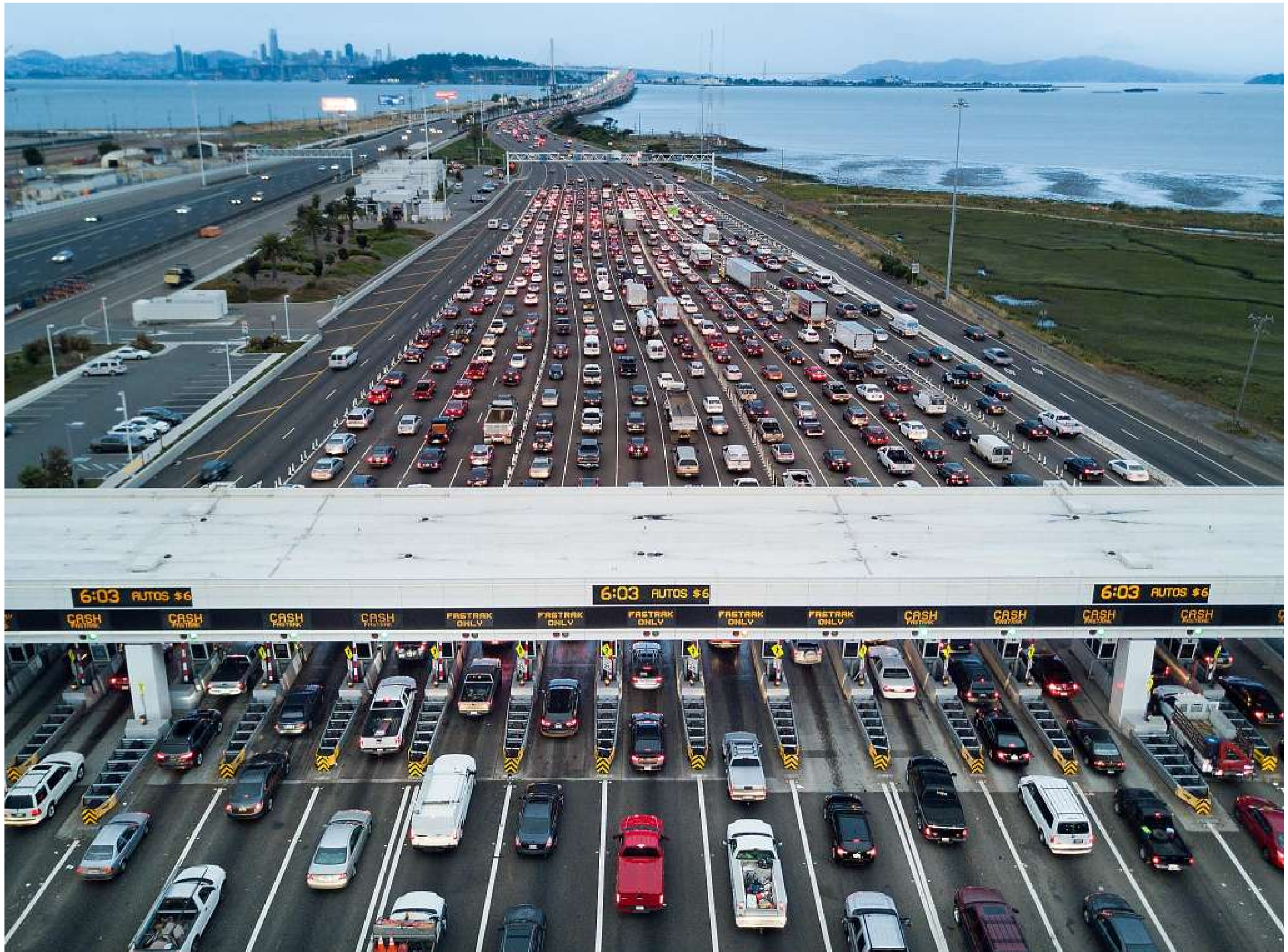
Self-driving cars



AVs publicly available as of July 2026:

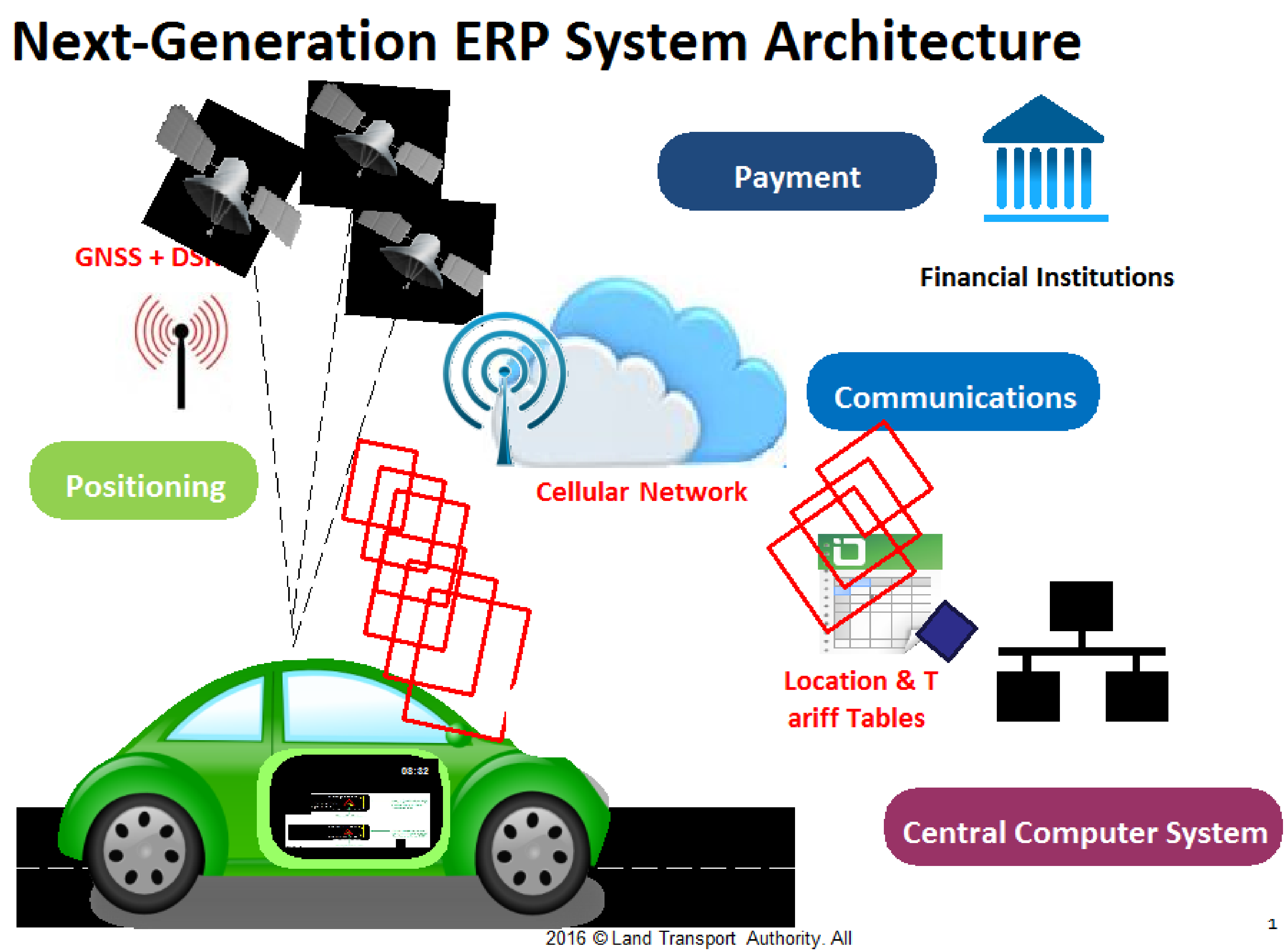
- Waymo (Google):
  - SF Bay Area, Phoenix, Los Angeles, Miami, Nashville, Orlando, Dallas, Houston, San Antonio, Austin, Atlanta
- Zoox (Amazon):
  - Las Vegas
- Apollo Go (Baidu):
  - Beijing, Chongqing, Wuhan, Shenzhen, Dubai
- WeRide:
  - Beijing, Guangzhou, Ordos, Shanghai, Abu Dhabi, Dubai
- Pony.ai:
  - Beijing, Guangzhou, Shenzhen

Tolls – current technology

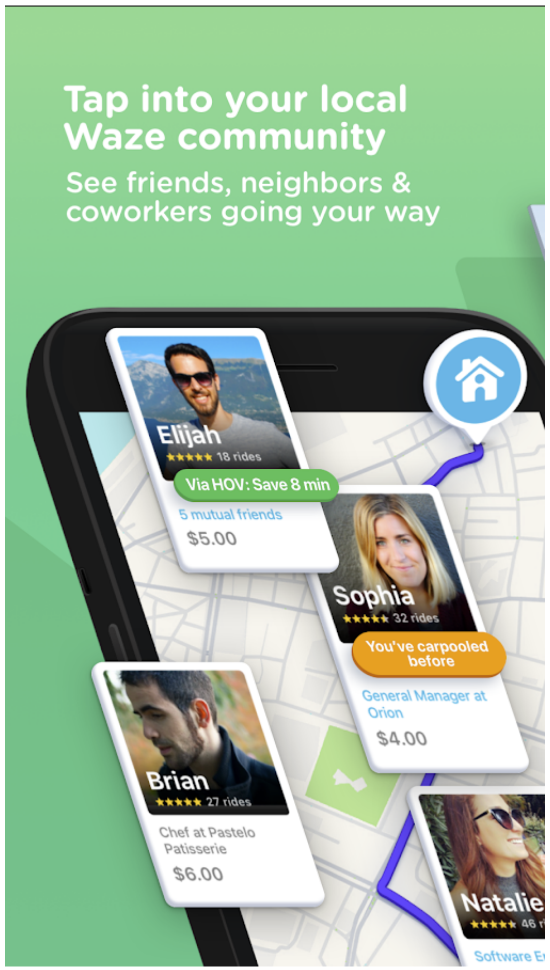


Tolls – current technology

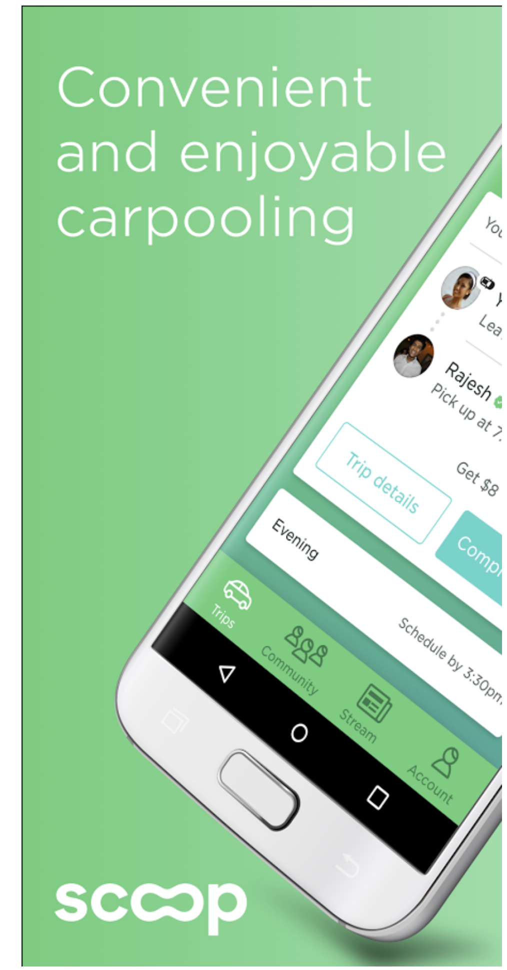
Tolls – coming soon



Carpooling


Tap into your local Waze community
See friends, neighbors & coworkers going your way
Elijah
18 rides
Via HOV: Save 8 min
5 mutual friends
$5.00
Sophia
32 rides
You've carpooled before
General Manager at Orion
$4.00
Brian
27 rides
Chef at Pastelo Patisserie
$6.00
Natalie


BlaBlaCar
Emma. BlaBla. Mum.


Convenient and enjoyable carpooling
Rajesh
Trip details
Evening
Trips
Community
Stream
Account
scoop

~~Car~~Vanpooling

The interplay among these three technologies

will be key for the future of transportation

Autonomous transportation and road pricing

- Lecture 2 and Small (1992): Equilibrium effect of self-driving cars on road demand during peak hours (in the absence of road pricing). Disutility of being stuck in traffic goes down $\Rightarrow$ demand goes up $\Rightarrow$ more traffic.

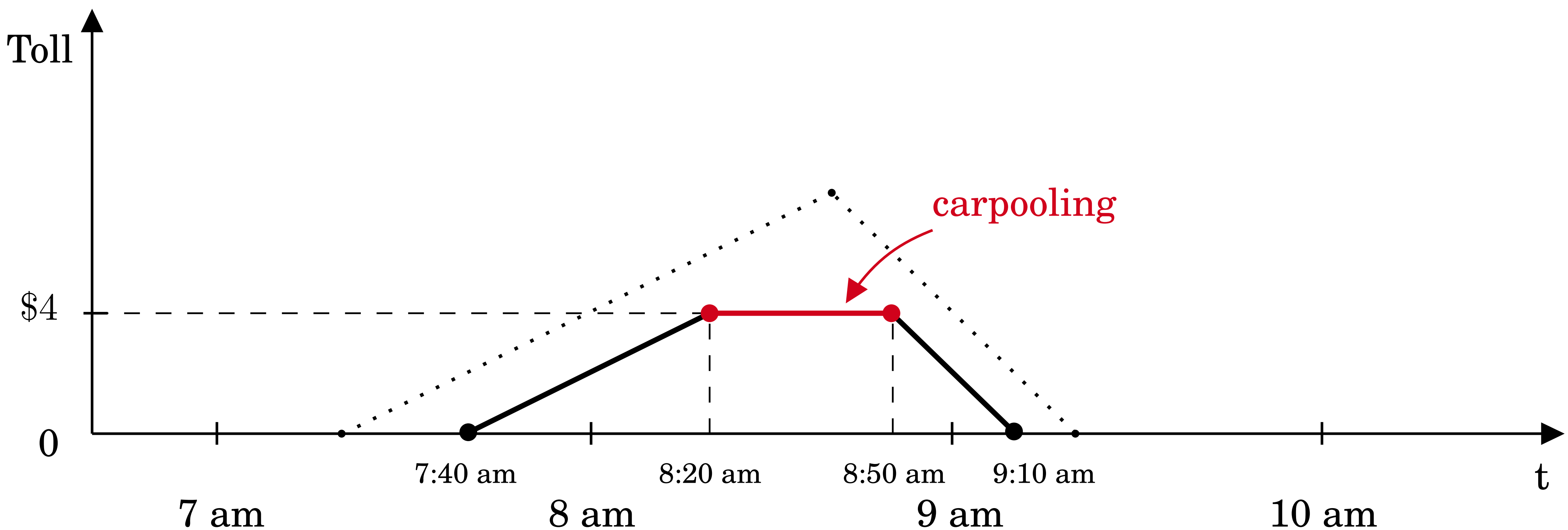
  > "A lot of people think that once you make cars autonomous that they'll be able to go faster and that will alleviate congestion and to some degree that will be true. But . . . the amount of driving that will occur will be much greater with shared autonomy and actually, traffic will get far worse."
  >
  > – Elon Musk, Tesla CEO (2017)

  In this case, intelligent road pricing becomes even more valuable.

- Logistical complementarity
  - Tolling infrastructure
  - User Interface

Carpooling and road pricing

- Lecture 2



Autonomous transportation and carpooling

With self-driving cars, “carpooling” will blur the line between solo driving and public transportation. Currently, the cost of drivers is a large component of public transit costs $\Rightarrow$ large vehicles, frequent stops, slow and often inconvenient service. Eliminating this cost $\Rightarrow$ smaller vehicles, fewer stops, door-to-door transit.

Carpooling in self-driving cars vs. carpooling now:

- Seamless coordination.

- No loss of flexibility.

- With several people sharing a ride, it does not have to be the case that the same person is the first one getting in the car and the last one getting out of it.

- Detours are much less costly: it costs much less to sit in the car as a passenger and read/sleep/work/watch movies while the car is making a five-minute detour than to drive during that time.

- No need to depend on a potentially unreliable carpool driver.

- Consistency of experience.

- One of the key challenges in creating a carpooling platform is getting to a critical mass where on-demand carpooling is reliable enough to be practical. With transportation services powered by self-driving cars, this issue is resolved automatically.

How could (should?) all of this be organized:

A mathematical model

Details in Ostrovsky and Schwarz (2018)

"Carpooling and the Economics of Self-Driving Cars"

Model

- Finite set of riders $m = 1, \ldots, M$.

- Finite set of road segments $s = 1, \ldots, S$. Each road segment s identifies both a physical road segment and a specific time interval.

- Each road segment s has an integer capacity $q_s > 0$.

- A trip is a feasible combination of one or more riders and one or more road segments. There is a finite number T of possible trips $t = 1, \ldots, T$.

- Each trip t has a physical cost $c(t)$ associated with it.

- Each rider m has a valuation for every trip t that involves him or her: $v_m(t)$.

- For each m, there exists an "outside option": a trip that involves only one rider (m) and no road segments, gives m the value of zero, and costs zero.

- An <u>assignment</u> is a set of trips A such that each rider is involved in exactly one trip in A.

- Assignment A is <u>feasible</u> if for each road segment s, the number of trips in A that include road segment s does not exceed its capacity q_s.

- The <u>social surplus</u> of assignment A is equal to the sum of the valuations of all the riders from the trips to which they are assigned in A minus the sum of the costs of the trips in A.

Monetary transfers and outcomes

- A non-negative vector $p \in \mathbb{R}^M$ specifies the price paid by each rider m.
- A non-negative vector $r \in \mathbb{R}^S$ specifies the tolls imposed by a regulator.
- An outcome is a triple (A, p, r) that specifies an assignment, the payments made by the riders, and the tolls imposed by the regulator.

An outcome (A, p, r) is

- feasible if the corresponding assignment A is feasible,

- budget-balanced if for every trip in A, the sum of prices paid by the riders for that trip is equal to the sum of the physical costs of the trip and the tolls on the road segments involved in it,

- stable if it is not possible to organize a trip t that makes all the riders involved in it strictly better off than they were under (A, p, r),

- market-clearing if for every road segment s that has “excess capacity” in A, the corresponding toll r_s is equal to zero.

Theorem 1:

If an outcome (A, p, r) *is feasible, stable, budget-balanced, and market-clearing, then assignment* A *has the highest possible social surplus among all feasible outcomes.*

Proof:

Take a feasible, stable, budget-balanced, and market-clearing outcome (A, p^A, r^A). Suppose feasible assignment B generates a higher social surplus than assignment A.

Take any trip t in B, and all riders m involved in t. By stability of (A, p^A, r^A), this coalition cannot benefit from organizing trip t by themselves, given prices p^A and tolls r^A in outcome A. Thus, summing the utilities of all riders involved in trip t:

$$\sum_{m \in t} \left(v_m(t_m^A) - p_m^A \right) \geq \left(\sum_{m \in t} v_m(t) \right) - c(t) - r^A(t). \tag{1}$$

Adding up equations (1) across all trips t in assignment B, we get

$$\sum_{m=1}^{M} v_m(t_m^A) - \sum_{m=1}^{M} p_m^A \geq \sum_{m=1}^{M} v_m(t_m^B) - \sum_{t \in B} c(t) - \sum_{s=1}^{S} r^A(s) k^B(s), \tag{2}$$

where $k^B(s)$ denotes how many trips in assignment B use segment s.

Since the outcome is by assumption budget-balanced, we have

$$\sum_{m=1}^{M} p_m^A = \sum_{t\in A} c(t) + \sum_{s=1}^{S} r^A(s)k^A(s).$$

Thus, equation (2) implies that

$$\sum_{m=1}^{M} v_m(t_m^A) - \sum_{t\in A} c(t) - \sum_{s=1}^{S} r^A(s)k^A(s) \geq \sum_{m=1}^{M} v_m(t_m^B) - \sum_{t\in B} c(t) - \sum_{s=1}^{S} r^A(s)k^B(s).$$

The social surplus of A is equal to $\sum_{m=1}^{M} v_m(t_m^A) - \sum_{t\in A} c(t)$, while the social surplus of B is equal to $\sum_{m=1}^{M} v_m(t_m^B) - \sum_{t\in B} c(t)$. To show that the former is greater than or equal to the latter, it is now sufficient to observe that

$$\sum_{s=1}^{S} r^A(s)k^A(s) \geq \sum_{s=1}^{S} r^A(s)k^B(s),$$

which follows from the market-clearing property of (A, p^A, r^A). ■

Issue: a feasible, stable, budget-balanced, market-clearing outcome may fail to exist. For example, suppose we have 101 riders and each car fits at most two passengers.

Approximation: allow trips to be "divisible". Formally, a "quasi-assignment" A is a measure on the set of trips such that overall, each rider is involved in measure 1 of trips. The definitions of an outcome and of feasibility, stability, budget-balancedness, and market-clearing extend naturally.

Theorem 2:

A feasible, stable, budget-balanced, and market-clearing quasi-outcome is guaranteed to exist. Any such quasi-outcome is socially efficient.

Idea of the proof:

Construct an auxiliary economy. Consumers in this economy are new agents called “trip organizers”. There are T types of those agents, one for each trip. There are 2 trip organizers of each type.

The goods are the M riders and S road segments from the original model. Price vector $\rho \in \mathbb{R}^{M+S} \geq 0$ assigns a price to every good in the economy.

Trip organizers can consume bundles of goods $b \in [0,1]^{M+S}$. Their utility is

$$\widetilde{V}_t(b) = \left(\sum_{m \in t} v_m(t) - c(t) \right) \times \min_{g \in t} b(g) - \rho^T b.$$

Competitive equilibrium: set of prices ρ and profile of consumption bundles b such that every consumer is optimizing and markets clear.

Existence of CE: For each ρ, take optimal bundles b, and add them up to get aggregate demand $D \in \mathbb{R}^{M+S}$. Let $Q \in \mathbb{R}^{M+S}$ denote the available supply of all items. Define the tâtonnement price adjustment function $\tau(\rho, D) = \max\{0, \rho + (D - Q)\}$.

Correspondence φ: for each ρ, take all possible D, and compute all possible $\tau(\rho, D)$.

Correspondence φ satisfies all the requirements of Kakutani's fixed-point theorem: $\varphi(\rho)$ is non-empty and convex for all ρ; and $\varphi(\rho)$ has a closed graph. Thus, correspondence φ has at least one fixed point. Any fixed point ρ^* of φ and a profile of consumer-optimal bundles with aggregate demand D^* such that $\rho^* = \tau(\rho^*, D^*)$ form a competitive equilibrium.

Last step: observe that in every CE, the utility of every consumer (trip organizer) is zero, because by construction, there is "excess supply" of trip organizers.

This observation allows us to go from a CE in the auxiliary market to a feasible, stable, budget-balanced, and market-clearing quasi-outcome:

- For each segment s, toll $r(s)$ is equal to the competitive equilibrium price $\rho^*(s)$.
- For each rider m, the utility $u(m)$ in the original market is set equal to $\rho^*(m)$.
- To construct the quasi-assignment G, take all riders m whose utility $u(m)$ is greater than zero and "read off" the volumes of various trips from these riders.

■

Takeaways

The government/transportation authority should price roads in such a way that there is no congestion, but at the same time not so high that some of the priced road segments are underutilized.

“Carpooling markets” will self-organize(*) as long as frictions are minimized.

The government/transportation authority should do whatever is in its power to reduce the disutility of carpooling and vanpooling and increase their convenience.

An observation about “car utilization”

A typical prediction about the world with self-driving cars: “The absurdity of our century-old, ad hoc approach to mobility is captured in one statistic: The utilization rate of automobiles in the U.S. is about 5%. For the remaining 95% of the time (23 hours), our cars just sit there, a slow, awful cash burn, like condos at the beach.” (*Wall Street Journal*, December 1, 2015)

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Our view: this point is a red herring

Why?

For empty cars, waiting for the next rider is cheap.

If cars die from usage (rather than old age) and the real interest rate is equal to zero, the increase in annual mileage yields no cost savings.

Numerical example

- Cars die after 200,000 miles or 15 years, whichever happens first
- New cars cost $50,000, real interest rate is 5% per year
- Gas/electricity, maintenance, remote assistance, and insurance are variable costs, $0.60 per mile

Annual mileage	5,000	10,000	15,000	20,000	40,000	80,000
Cost per mile	$1.55	$1.07	$0.94	$0.92	$0.88	$0.87

Other implications

- Limited cost reduction of using "Transportation-as-a-serivce" vs. personal ownership for many people (e.g., those driving 15,000 miles per year or more and living in areas with low parking costs), so personal self-driving cars may easily co-exist with various transportation services. Our framework accommodates this possibility.

- "High utilization" in the form of putting more people in the car (carpooling) will be much more important than "high utilization" in the form of putting more miles on a car. E.g., potential cost savings from doubling car occupancy are an order of magnitude higher than the savings from doubling the amount of time a car spends on the road per day.